T4. The nuclear past of Uzbekistan

6%									
4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	∑
2	3	3	2	2	2	2	2	4	22

Natural uranium consists primarily of two isotopes, ^{235}U and ^{238}U. ^{235}U is responsible for sustaining nuclear fission reactions.

4.1 **Calculate** the atomic abundance of ^{235}U in natural uranium, assuming it consists only of isotopes ^{235}U (235.04 a.u.) and ^{238}U (238.05 a.u.). 2.0 pt

SOLUTION:

Using the average atomic mass of uranium from the Periodic Table as 238.03 a.u., and isotope masses as 235.04 a.u. for $^{235}\mathrm{U}$ and 238.05 a.u. for $^{238}\mathrm{U}$, we can calculate the atomic abundance, x, of $^{235}\mathrm{U}$.

Equation:

$$238.03 = x \times 235.04 + (1 - x) \times 238.05 \qquad (\mathbf{1})$$
$$238.03 = 235.04x + 238.05 - 238.05x$$
$$238.03 - 238.05 = x\,(235.04 - 238.05)$$
$$-0.02 = x\,(-3.01)$$
$$x = \frac{-0.02}{-3.01} = 0.00664 = 0.66\%$$

The atomic abundance of $^{235}\mathrm{U}$ in natural uranium is approximately 0.66%, which is a slightly lower estimate compared to the commonly accepted value of about 0.72%, influenced by rounding and exact values used.

The atomic abundance of $^{235}\mathrm{U}$ is $\underline{0.66\%}$

2 points total: 1 point for equation **(1)**;

1 point for the correct answer.

The main reaction occurring in nuclear power plants is the reaction where ^{235}U absorbs a neutron and undergoes fission, releasing energy and producing three additional neutrons, thus sustaining a chain reaction.

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow \ldots + 3^{1}_{0}\mathrm{n}$$

Neutrons that induce the chain reaction are produced by several ways:

a) Reaction of ^{11}B with α-particles inside the reactor core

b) Interaction of γ rays with ^{2}D nuclei present in the reactor coolant water

c) Reaction of ^{9}Be with α-particles produced in an externally installed neutron source.

4.2 **Write** the nuclear reaction equations for **(a)**, **(b)**, and **(c)**. 3.0 pt

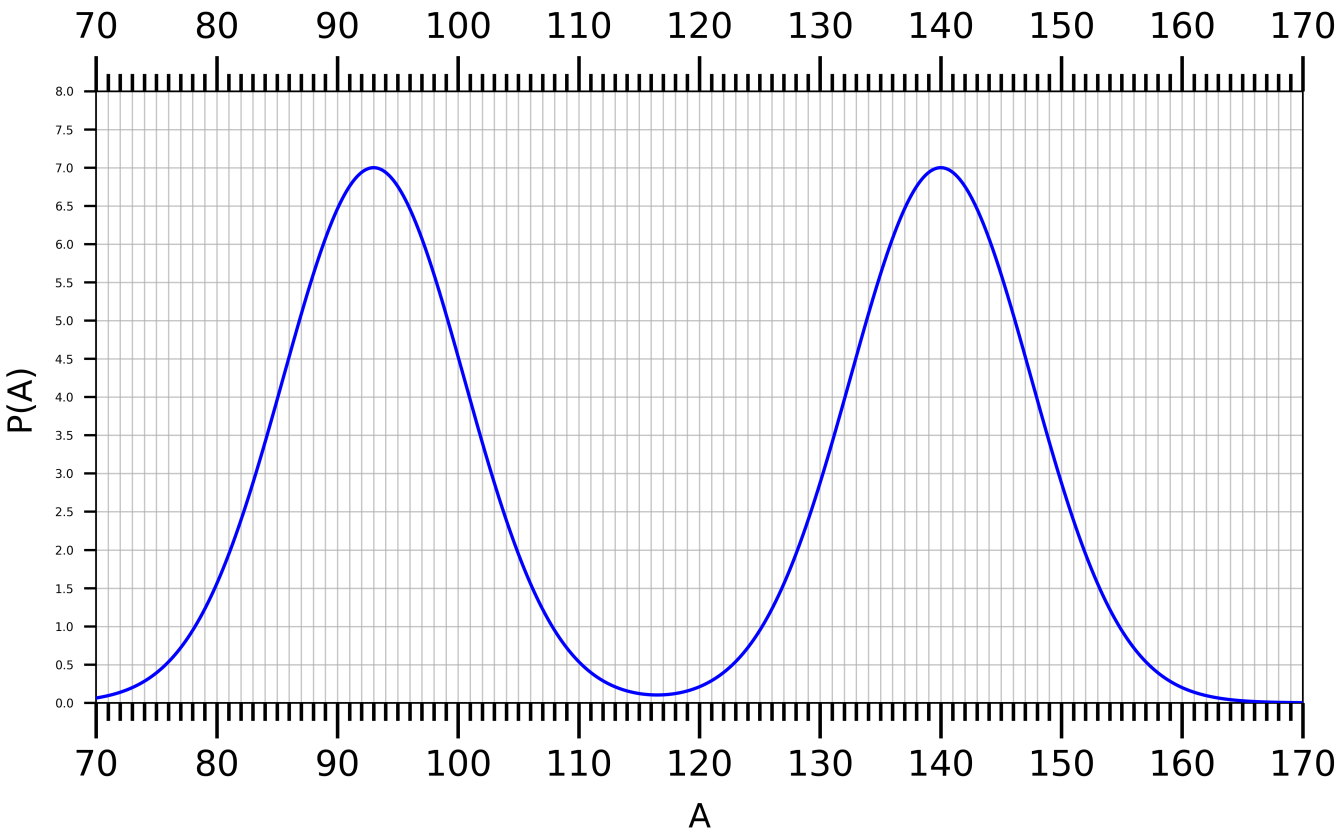
SOLUTION:

Reaction (a): ${}^{11}_{5}\mathrm{B} + {}^{4}_{2}\alpha \rightarrow {}^{14}_{7}\mathrm{N} + {}^{1}_{0}\mathrm{n}$

Reaction (b): $\gamma + {}^{2}_{1}\mathrm{D} \rightarrow {}^{1}_{1}\mathrm{H} + {}^{1}_{0}\mathrm{n}$

Reaction (c): ${}^{9}_{4}\mathrm{Be} + {}^{4}_{2}\alpha \rightarrow {}^{12}_{6}\mathrm{C} + {}^{1}_{0}\mathrm{n}$

3 points total: 1 point for each full and correct equation.



The graph above displays the percentage yield, *P(A),* of fission products by mass number, *A,* resulting from ^{235}U fission.

4.3 **Write** the main nuclear fission equation for splitting of ^{235}U upon neutron absorption. The pair of elements formed in this reaction are in the same group of Periodic Table. 3.0 pt

SOLUTION:

Let us presume that total atomic mass of elements in the fission products is $\sum A = 235+1-3 = 233$ There are two maxima on the graph at $A = 93$ and $A = 140$, which sum up to 233. Thus, there are only two fission products beside neutrons considered in the question. To find the atomic numbers we use conservation of charge and come to equation: $x + y = 92$ with x and y denoting atomic numbers of elements. These elements are in the same group, the intervals between them may be 2, 8, 18, 32, and 50. Let's solve the equation for those parameters:

$x + (x + a) = 92 \text{ where } a = 2, 8, 18, 32, 50$

$x = 45, y = 47 \ (Rh,\ Ag, a = 2)$

$x = 42,\ y = 50 \ (Mo, Sn, a = 8)$

$x = 37, y = 55 \ (Rb, Cs, a = 18)$

$x = 30, y = 62 \ (Zn, Sm, a = 32)$

$x = 21, y = 71 \ (Sc, Lu, a = 50)$

Technically, Sc-Lu pair might do it, but there is no evidence for such high mass number isotope of scandium ($A = 93$), so the only reasonable answer is Rb and Cs.

Equation of nuclear fission reaction: $^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{93}_{37}\text{Rb} + ^{140}_{55}\text{Cs} + 3^{1}_{0}\text{n}$

3 points total:

0.6 point for each correct atomic mass of two elements (1.2 points). If the student provides the sum of the atomic masses of two elements they will also be given 1.2 points.

0.6 point for each correct element (1.2 points).

0.6 point for nuclear fission reaction equation.

4.4 **Calculate** the energy released in this reaction (ΔE, MeV), if the binding energy $BE(^{235}\text{U})$ = 7.59 MeV/nucleon and average binding energy for fission products $BE(\text{fis.})$ = 8.45 MeV/nucleon. *Neglect* the binding energy of free neutrons. 2.0 pt

SOLUTION:

$\Delta E = 8.45 \times 233 - 7.59 \times 235 = 185.20 \text{ MeV}$

Released energy: 185.2 MeV

2 points for the correct answer and relevant calculations; 1 point may be given for the right formula but the wrong answer.

Fission neutrons are produced with an average energy of 2 MeV and slow down through many scatterings with nuclei. After several collisions, their speed decreases until their average kinetic energy matches that of the surrounding atoms or molecules.

The **logarithmic energy decrement,** ξ , shows the average decrease per collision in the logarithm of the neutron energy:

$$\xi = \ln\left(\frac{E_{initial}}{E_{final}}\right)$$

$E_{initial}$ and E_{final} are the initial and final neutron energies per collision.

ξ is a constant for each type of material and does not depend on the initial neutron energy.

4.5 **Determine** the average number of collisions required, n_c, to slow a neutron from 2 MeV to 0.012 eV, using water as the moderator. ξ (water)=0.948 — 2.0 pt

SOLUTION:

$$n_c = \frac{\ln\left(\frac{E_{initial}}{E_{final}}\right)}{\xi} = 19.97 \approx 20$$

2 points for the correct answer 20.

1 point may be given for the right formula but the wrong answer.

In 1963, the *Urtabulak* gas field suffered an uncontrolled natural gas well blowout, releasing enormous volumes of burning methane.

4.6 **Calculate** $\Delta_r H^\circ_{298}$ (kJ mol^{-1}) for one mole of methane combustion reaction at 298 K. **Assume** all species are gaseous. Use the thermodynamic data below — 2.0 pt

SOLUTION:

Balanced reaction: $CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O(g)$

$\Delta_r H^\circ_{298} = [\Delta_f H^\circ(CO_2) + 2\Delta_f H^\circ(H_2O)] - [\Delta_f H^\circ(CH_4) + 2\Delta_f H^\circ(O_2)]$

$\Delta_r H^\circ_{298} = ([-393.5 + 2(-241.8)] - [-74.8 + 0])\mathrm{kJ\ mol^{-1}} = -802.3\ \mathrm{kJ\ mol^{-1}}$

Molar enthalpy of methane combustion at 298 K, $\Delta_r H_{298} = -802.3\ \mathrm{kJ\ mol^{-1}}$

2 points for the correct answer; 1 point if the answer is incorrect but the calculation of difference in enthalpies of products and reagents exists.

$\Delta_f H^\circ_{298}(CH_4)$	-74.8 kJ mol^{-1}
$\Delta_f H^\circ_{298}(H_2O, gas)$	-241.8 kJ mol^{-1}
$\Delta_f H^\circ_{298}(CO_2)$	-393.5 kJ mol^{-1}
$C_P(CH_4)$	35 J mol^{-1} K^{-1}
$C_P(H_2O, gas)$	34 J mol^{-1} K^{-1}
$C_P(O_2)$	29 J mol^{-1} K^{-1}
$C_P(CO_2)$	37 J mol^{-1} K^{-1}

If you did not get an answer for 4.6, use $\Delta_r H^\circ_{298} = -750\ kJ\ mol^{-1}$ for further calculations.

4.7 **Calculate** $\Delta_r H_{2000}$ (kJ mol^{-1}) per mole of methane combustion reaction at 2000 K. **Assume** all species are gaseous. 2.0 pt

SOLUTION:

Assume constant heat capacities.

$\Delta H(T) = \Delta H(298\ \ K) + \Delta C_p(T - 298\ \ K)$

$\Delta C_p = [C_p(CO_2) + 2C_p(H_2O)] - [C_p(CH_4) + 2C_p(O_2)]$

$\Delta C_p = [37 + 2(34)] - [35 + 2(29)] = 12\ \ \text{J mol}^{-1}\ \text{K}^{-1}$

$\Delta_r H^{\circ}_{2000} = -802.3\ \text{kJ mol}^{-1} + \frac{(12\text{Jmol}^{-1}\text{K}^{-1})(1702\ \ K)}{1000} = -781.876\ \text{kJ}\ \ \text{mol}^{-1}$

$\Delta_r H^{\circ}_{2000} = -781.9\ \ \text{kJ mol}^{-1}$

For a given value of $\Delta_r H^{\circ}_{298} = -750\ \text{kJ/mol}$ alternative answer will be:

$\Delta_r H_{2000} = -781.9\ \ kJ\ \ mol^{-1}$

$\Delta_r H^{\circ}_{2000} = -750\ \ \text{kJ mol}^{-1} + \frac{(12\ \ \text{J mol}^{-1}\text{K}^{-1})(1702\ \text{K})}{1000} = -729.576\ \ \text{kJ mol}^{-1}$

$\Delta_r H_{2000} = -729.6\ \ \text{kJ mol}^{-1}$

2 points total:

1 point for the correct heat capacity change calculation.

1 point for the correct enthalpy calculation at 2000 K.

If you did not get an answer for 4.7, use $\Delta_r H_{2000} = -700\ kJ\ \ mol^{-1}$ for further calculations.

Assume that methane is flowing out the well at 101.325 kPa and 298 K with volumetric flow Q = 2.2 × 10^5 m^3 per day before it catches fire.

4.8 **Calculate** the total energy released per day, E, (in J day^{-1}) by complete isothermic combustion of methane at T = 2000 K 2.0 pt

SOLUTION:

Using the flow as a gas volume measured at 1 atm and 298 K:

$n = \frac{PV}{RT} = \frac{101325\ \text{Pa}\times 2.2\times 10^5\ \text{m}^3}{8.314\ \text{J mol}^{-1}\ \text{K}^{-1}\times 298\ \text{K}} = 9.00 \times 10^6$ mol day^{-1}

Total combustion energy released daily = 7.04×10^{12} J day^{-1}

For provided $\Delta_r H_{2000} = -700$ kJ mol^{-1} the answer = 6.30×10^{12} J day^{-1}

2 points for the correct answer.

1 point if the answer is incorrect but the relevant calculation exists.

The *Urtabulak* gas leak was ended by detonating a 30-kiloton in TNT equivalent underground nuclear explosion that collapsed the reservoir and sealed the leak. 1 ton of TNT equivalent is 4.184 GJ of energy.

4.9 Using the value from **4.4**, **calculate** the total number of fissions, *TN*, during the nuclear explosion. **Calculate** the mass, *m*, of enriched uranium used in the explosion (in kg), assuming it contained 90% by mass of the ^{235}U isotope, and 33% of the ^{235}U underwent fission. 4.0 pt

If you did not get an answer for 4.4, use $\Delta E = 200$ MeV.

SOLUTION:

Explosion energy $E = 30.0 \times 10^3$ ton $\times 4.184$ GJ ton^{-1} $= 1.2552 \times 10^{14}$ J

Using energy per fission from **4.4**:

185.2 MeV $\times 1.602 \times 10^{-13}$ J MeV^{-1} $= 2.967 \times 10^{-11}$ J

$TN = \frac{E}{E_{\text{fission}}} = \frac{1.2552\times10^{14}}{2.967\times10^{-11}} = 4.23 \times 10^{24}$ fissions

$n(^{235}U) = \frac{N}{N_A} = \frac{4.27\times10^{24}}{6.022\times10^{23}\ \text{mol}^{-1}} = 7.03$ mol

$m = 7.03$ mol $\times 235.04$ g mol^{-1} $= 1.65 \times 10^3$ g = 1.65 kg

Accounting that enriched uranium contains 90% of ^{235}U isotope and only 33% is undergoing decay process:

$\frac{1.67}{0.9\times0.33} = 5.56$ kg

$TN = 4.23 \times 10^{24}$ fissions

$m = 5.56$ kg

4 points total: 2 points for the correct number of fissions; 1 point if the answer is incorrect but the relevant calculation exists. 2 points for the correct mass of enriched uranium; 1 point if the answer is incorrect but the relevant calculation exists.